

31
 40

Section Two (Calculator free)

Mark Available:

10
 17

Time Allowed: 25 minutes

Student's Name: Chu Minh Dong

(7 marks)

Question 1

(2 marks)

(a) Simplify $\sqrt{4^{-5}}$.

$(4^{-5})^{\frac{1}{2}}$ ✓ ①
 $4^{-\frac{5}{2}}$?

(b) Write the value of xy in scientific notation when $x = 2.5 \times 10^3$ and $y = 5 \times 10^{-7}$.

(2 marks)

~~2.5 x 5~~
 $2.5 \times 5 \times 10^3 \times 10^{-7}$
 12.5×10^{-4} ✓ ①
 1.25×10^{-3} ✓ ①

(c) Determine the value of n given that $9^{n+1} = \sqrt{27}$.

(3 marks)

$2n = 3$
 $n = \frac{3}{2}$

$(3^2)^{n+1} = 27^{\frac{1}{2}}$
 $3^{2n+2} = (3^3)^{\frac{1}{2}}$ ✓ ①
 $2n+2 = \frac{3}{2}$ ✓ ①
 $2n = \frac{3}{2} - 2$
 $2n = \frac{3}{2} - \frac{4}{2}$
 $2n = -\frac{1}{2}$

$2n = -\frac{1}{2}$ ✓ ①
 $n = -\frac{1}{4}$ ✓ ①

$$\frac{2^{-1} a^{\frac{1}{3}} b^{-\frac{2}{3}}}{3^{-1} a^{\frac{2}{3}} b^{\frac{1}{3}}} \quad \checkmark \textcircled{1}$$

(10 marks)

Question 2

a. Simplify the following, leaving all indices positive.

$$(1) \left(\frac{8a^{-1}b^2}{27a^2b^{-1}} \right)^{-\frac{1}{3}} = \left(\frac{2^3 a^{-1} b^2}{3^3 a^2 b^{-1}} \right)^{-\frac{1}{3}} = \frac{3a}{2b} \quad \checkmark \textcircled{1}$$

(2 marks)

~~$$\frac{2a^{-\frac{1}{3}} b^{\frac{2}{3}}}{3 a^{\frac{2}{3}} b^{-\frac{1}{3}}} = \frac{2b^{\frac{2}{3}} \times b^{\frac{1}{3}}}{3 a^{\frac{2}{3}} \times a^{\frac{1}{3}}} = \frac{2b}{3a}$$~~

$$(2) \frac{3^n + 3^{n+2}}{3^{n-1}}$$

(2 marks)

~~$$\frac{3^n + 3^{n+2}}{3^{n-1}} = \frac{3^n(3+3^2)}{3^{n-1}} = \frac{3+3^2}{3^{-1}} = \frac{3+9+3}{1} = 15$$~~
$$\frac{3^n(3+3^2)}{3^{n-1}}$$

b. Solve for x.

$$4^x = \sqrt{8}$$

(2 marks)

~~$$(2^2)^{\frac{1}{2}x} = 8^{\frac{1}{2}}$$

$$(2^4)^{\frac{1}{2}x} = (2^3)^{\frac{1}{2}}$$

$$2^{\frac{2}{2}x} = \frac{3}{2}$$~~

~~$$\frac{2}{x} = \frac{3}{2} \quad \checkmark \textcircled{1}$$~~
~~$$x = \frac{4}{3}$$~~

~~$$x = 1.5$$

$$x = \frac{4}{3}$$~~

(c) Solve algebraically for x.

(2 marks)

$$16^{\frac{x-5}{2}} = \sqrt[3]{64}$$

~~$$(4^2)^{\frac{x-5}{2}} = (64)^{\frac{1}{3}}$$

$$4^{\frac{2x-5}{2}} = (4^3)^{\frac{1}{3}}$$~~

~~$$\frac{2x-5}{2} = 1$$~~

~~$$2x-5 = 2$$~~

~~$$2x = 7$$~~

~~$$x = \frac{7}{2}$$~~

$\textcircled{1}$ c.f

~~$$\frac{2}{2} = \frac{16}{24}$$~~

~~$$4 = \frac{2x-5}{2} \quad 2 = 4^1$$~~

(d) Given that $3^x = 5$, determine the value of 9^{x+1} .

(2 marks)

$$3^x = 5$$

$$(3^2)^{x+1} = 3^{2x+2}$$

$$3^{2x+2} = 3^{2x+1} \cdot 3$$

$$3^{2x+2} = 3^{2x+1} \cdot 3$$

$$3^4 = 3, 9, 27, 81$$

End of section 1

Saigon International College
Department of Mathematics and Science
Semester 2, 2022
Year 11 ATAR Mathematics Methods
Test 4
(Indices, exponential functions)

Section Two (Calculator assumed)

21

Mark Available: 23

Time Allowed: 45 minutes

Student's Name: ... *Chu Minh Duy* ...

Question 3

(5 marks)

The area of forest in Methodland is estimated to be decreasing at a rate of 12% per year. In January 2010 the area of the forest was 275 km².

- i. Write down an equation in the form $A = A_0k^t$, where A_0 is the initial area, t is the time in years, after 2010 and k is a fixed constant. [2 marks]

$$A = 275 \times 0.88^t$$

- ii. What is the area of the forest expected to be in 2020? [1 mark]

$$275 \times 0.88^{10}$$

$$76.58 \text{ km}^2$$

- iii. In what year is the area of the forest expected to be 50 km²? [2 marks]

$$50 = 275 \times 0.88^t$$

$$t = 19.3$$

beginning of year 2024 rounded

1 or April of 2023 to be exact

Question 4

(6 marks)

- (a) Determine the solution(s), if any exist, when $3^{x-1} = 6$. Give your answer correct to one decimal place. (2 marks)

$$3^{x-1} = 6$$

one solution

$$x = 2.6$$

✓ (2)

- (b) (i) Determine the coordinates of the points of intersection of the functions $y = 3^{x-1}$ and $y = x+1$. (2 marks)

$$y = 3^{x-1} \quad \text{and} \quad y = x+1$$

$$x+1 = 3^{x-1}$$

$$x = -0.8721, 0.12786$$

$$\boxed{(-0.8721, 0.12786)}$$

✓ (1)

$$x = 2$$

$$y = 2+1$$

$$y = 3$$

$$\boxed{(2, 3)}$$

✓ (1)

- (ii) Calculate the distance between the points of intersection in correct to 2 significant figures. (2 marks)

$$3 - 0.12786 = 2.8721 \quad \text{or} \quad 3 - 0.12$$

$$2 + 0.87213 = 2.87213 \quad \text{or} \quad 2 - 0.87 = 2.87$$



2
 answer: 4.04 ✓ (1)

≈ 4.0
 (2. sig fig)

Question 5

(6 marks)

(a) Sleeping Beauty slept for 200 years. She had \$2 in the bank where she started sleeping. The interest rate during those years remained at a constant 5.75% compounded annually.

i. How much would she have in her bank account when she wake up? (2 marks)

$$2 \times 1.0575^t$$

$$\$ 143,589.72 \text{ (very rich)} \quad (2)$$

ii. How long would she need to sleep if she wanted to wake up with at least \$1,000,000 in the bank? (2 marks)

$$1000000 = 2 \times 1.0575^x \Rightarrow 235 \text{ years} \quad (2)$$

(b) The population of a mining town in northern Australia decreases from 185430 people to 105216 over a period of 3 years. What was the constant percentage rate of decrease in the population per year over that period? (2 marks)

$$105216 = 185430 \times r^3 \quad \checkmark$$

$$r = \frac{105216}{185430} \Rightarrow 0.827 \quad \checkmark$$

$$0.827 \quad (1)$$

17% decrease each year (1)

Question 6

(6 marks)

The cost, C dollars, for a gigabyte of computer memory between the end of year 2005 ($t = 0$) and the end of year 2015 ($t = 10$) can be modelled by the equation $C = 13.5(0.75)^t$.

(a) Calculate C at the end of year 2010.

(1 mark)

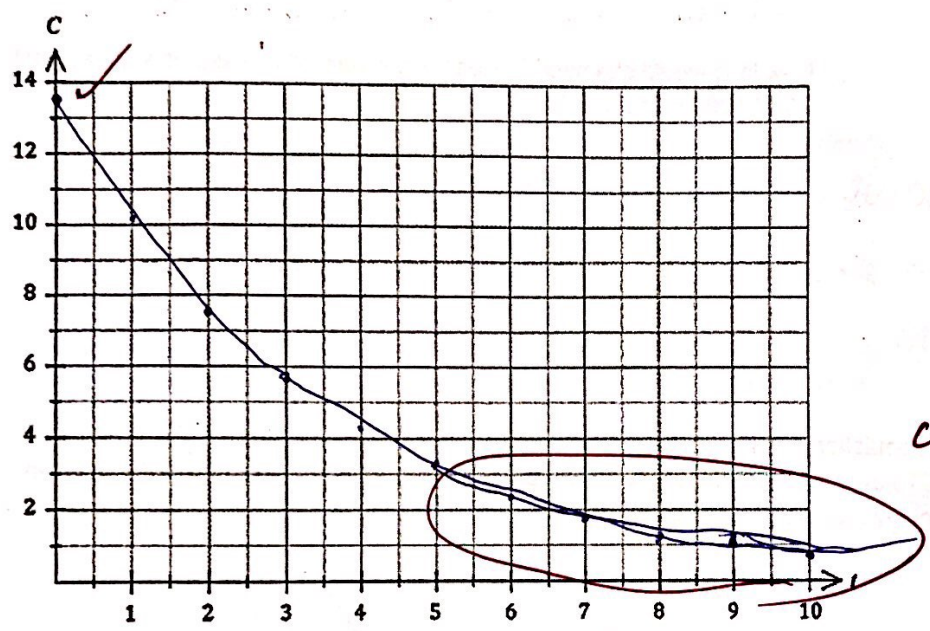
$$C = 13.5 (0.75)^5$$

$$C = 49.20$$

~~CAS~~ ①

(b) Draw the graph of C against t on the axes below.

(3 marks)



arks)

- (c) Assuming that the model continues to be valid, during which year will the cost of computer memory fall below 20 cents per gigabyte? (2 marks)

$$0.2 = 13.5 \times 0.75^x \quad \checkmark$$

$$x = 14.6 \text{ years} \quad \checkmark \quad \textcircled{1}$$

By the year 2019 (Around July)
or the year 2020 rounded $\checkmark \quad \textcircled{1}$

End of section 2